Measurement of coefficient of restitution made easy

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Abstract

We present a simple activity that permits students to determine the coefficient of restitution of bouncing balls using only a stopwatch, a metre stick and graphical analysis. The experiment emphasizes that simple models, in combination with careful attention to how students make measurements, can lead to good results in a straightforward way.

Introduction

The coefficient of restitution (COR, represented by the letter e) is a measure of how elastic a collision is. A value of $e = 1$ represents a perfectly elastic collision, and a value of $e = 0$ indicates a totally inelastic interaction. Measuring the COR has been discussed in many previous articles about physics teaching [1–10] using balls of various types. Several of these articles have discussed the use of high-speed data collection methods to 'listen' to the bouncing and extract the COR, or detailed analysis with high-speed cameras and force sensors.

As long as a ball bounces from a massive and rigid surface, it is appropriate to discuss the COR of the ball, although there are actually two objects involved in the collision. These previous studies have shown that the COR is not really a constant but does depend slightly on velocity, and thus on the original release height in the case considered here of a ball held at rest and dropped without spin. However, these variations of the COR are minor, and we will assume that they are negligible in the experiment discussed below. Whereas many COR activities require students to measure the rebound height of balls, our activity enables students to determine the COR by measuring time.

Student activity

The activity is quite simple. Students drop a ball vertically from rest at a known height (h_0) as measured with a metre stick and use a stopwatch to measure the total time (t_{total}) from release until the ball stops bouncing. The major uncertainties in the experiment come from the release of the ball (which may induce unwanted spinning) and the quality of the horizontal surface on which the ball bounces. The surface needs to be horizontal, massive, hard and smooth. Otherwise the ball will not bounce vertically but will 'wander off', or too much energy will be transferred to the surface, resulting in an artificially low value of the COR. This makes an excellent 'take home' or outside activity for students of various ages and skill levels. Students simply record sets of t_{total} versus h_0 data with various balls and then bring their data into the class for analysis, or do the graphing at home if they have the ability to work independently.

Extracting the COR from the data

Assuming no air resistance and only vertical motion, we can use conservation of mechanical

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energy

$$\tfrac{1}{2}mv^2 = mgh \tag{1}$$

to see that the definition of the COR in terms of the ratio of the final velocity after a collision (v_f) to that before the collision (v_i) reduces to the square root of the ratio of the two relevant heights in our case:

$$e = v_f/v_i = (h_f/h_i)^{1/2} \tag{2}$$

where the acceleration due to gravity (g) and the mass of the ball (m) divide out. The total time from the release until the ball stops bouncing is simply the sum

$$t_{total} = t_0 + t_1 + t_2 + t_3 + \ldots \tag{3}$$

with the time for the initial descent

$$t_0 = (2h_0/g)^{1/2} \tag{4}$$

and the subsequent bounces, including round trips, given by

$$\begin{aligned} t_1 &= 2(2h_1/g)^{1/2} \\ &= 2(h_1/h_0)^{1/2}(2h_0/g)^{1/2} \\ &= 2e(2h_0/g)^{1/2} \end{aligned} \tag{5}$$

$$\begin{aligned} t_2 &= 2(2h_2/g)^{1/2} \\ &= 2(h_2/h_1)^{1/2}(h_1/h_0)^{1/2}(2h_0/g)^{1/2} \\ &= 2e^2(2h_0/g)^{1/2} \end{aligned} \tag{6}$$

and so forth, where the factor of 2 in front of each term is to account for the round trip time for each bounce. Note that we are neglecting the time of contact of the ball with the surface and also assuming that the COR is constant in this simple treatment. Substituting expressions (4)–(6) and others that logically follow into (3), we find

$$t_{total} = (2h_0/g)^{1/2}[1 + 2e + 2e^2 + 2e^3 + \ldots] \tag{7}$$

which can be rewritten slightly by adding and subtracting 1 as

$$t_{total} = (2h_0/g)^{1/2}[2+2e+2e^2+2e^3+\ldots-1] \tag{8}$$

which suggests the form

$$t_{total} = (2h_0/g)^{1/2}[2\{1+e+e^2+e^3+\ldots\}-1]. \tag{9}$$

The expression inside the curly brackets { } is a geometrical series that can be summed to yield [11]

$$t_{total} = (2h_0/g)^{1/2}[2\{1/(1-e)\} - 1]. \tag{10}$$

So we see that the total time is proportional to the square root of the original height of the ball before release. Expecting this behaviour, plotting t_{total} versus $h_0^{1/2}$ should yield a straight line with a slope (S) seen from equation (10) to be

$$S = (2/g)^{1/2}\frac{1+e}{1-e} \tag{11}$$

where we have rewritten (10) using the least common denominator. Note that as the COR approaches zero, the total time approaches $(2h_0/g)^{1/2}$, and as the COR gets close to 1, the total time tends to infinity, as expected. Finally, for ease of use, this can be inverted to arrive at

$$e = \frac{S(g/2)^{1/2} - 1}{S(g/2)^{1/2} + 1}. \tag{12}$$

Except for the geometrical series limit, which is accessible only to advanced students, those who can do only algebra can still understand and use this simple model treatment of the system. All that is required is the assumption of conservation of energy, the neglect of a few minor items like air resistance and variations of e with temperature, humidity and velocity, and that the teacher provides the algebraic guidance and correct substitution for the series approximation. Combined with the simplicity of the actual experiment, this is an activity that should be useful to many physics instructors and teachers.

We would like to note that the mathematical approach presented in [8], which we discovered during the preparation of this manuscript, is similar to ours, but that our experimental method is very different. Also, if students and teachers do not wish to get too involved in the algebra and the geometrical series, dimensional analysis can be used to show that the total time must be proportional to $(h_0/g)^{1/2}$. The proportionality factor must represent the elastic properties of the ball, which we show to be the slope defined in equation (11). The more elastic the ball, the longer it bounces, the closer the COR gets to 1, and the steeper the slope of the graph of t_{total} versus h_0, all making physical sense.

Representative results

Figure 1 shows some results generated by students from one of our classes who carefully did this activity at home with balls that they happened to

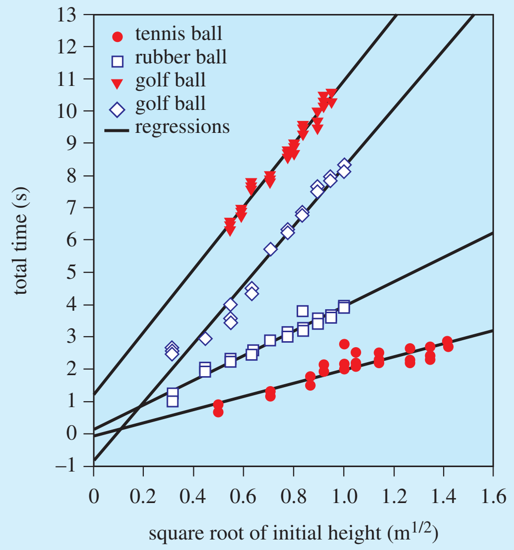

Figure 1. Data generated by some of our students during a 'take home' lab with balls that they had available.

have available. This group of students was actually composed of future middle school teachers. The least-squares regression lines and the R^2 values from the computer-generated fits (SigmaPlot) are shown in table 1, along with the COR determined using equation (12). The agreement of the slopes from two independent experiments with golf balls shows very good self-consistency. It should be pointed out that we have not forced the fits to pass through the origin, as would be expected from the model discussed above. Non-zero y-intercepts represent systematic errors that can be minimized by attentive students. Finally, if students are not familiar with computerized data analysis, these data could even be analysed with graph paper by hand.

Students need to be careful with how they drop the ball so as not to induce spin, use reproducible timing techniques, choose a surface to bounce the ball on that is hard and smooth, and conduct experiments over a wide range of initial heights that they can access safely. Even with these experimental details in mind, the activity is simple and fun to perform, and yields results that are consistent with expectations.

Table 1. Results from some of our students who did this activity at home.

	Regression slope ($s\,m^{-1/2}$)	R^2 value	Average COR
Tennis ball	2.03	0.842	0.636
Rubber ball	3.82	0.973	0.788
Golf ball	9.12	0.972	0.906
Golf ball	9.76	0.976	0.911

Summary

In summary, this experiment is easily performed by students with a variety of skill levels without the need for special equipment. It emphasizes careful measurements and graphical analysis, and uses a simple model to determine the coefficient of restitution of bouncing balls.

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