Circuito diferenciador

$X_1 = i(t)$

$\dot{X}_1 = -\frac{1}{RC} X_1(t) + u$ $\Rightarrow$ $\dot{X}_1 = -\frac{1}{RC} X_1 + u$

$y = -\frac{1}{RC} X_1 + u$

$y = R\dot{X}_1 \frac{1}{R} = \dot{X}_1 = -\frac{1}{RC} \cdot X_1 + u$

$\lambda = -\frac{1}{RC}$ $\rightarrow$ $Z(t) = e^{-\frac{1}{RC}t}$

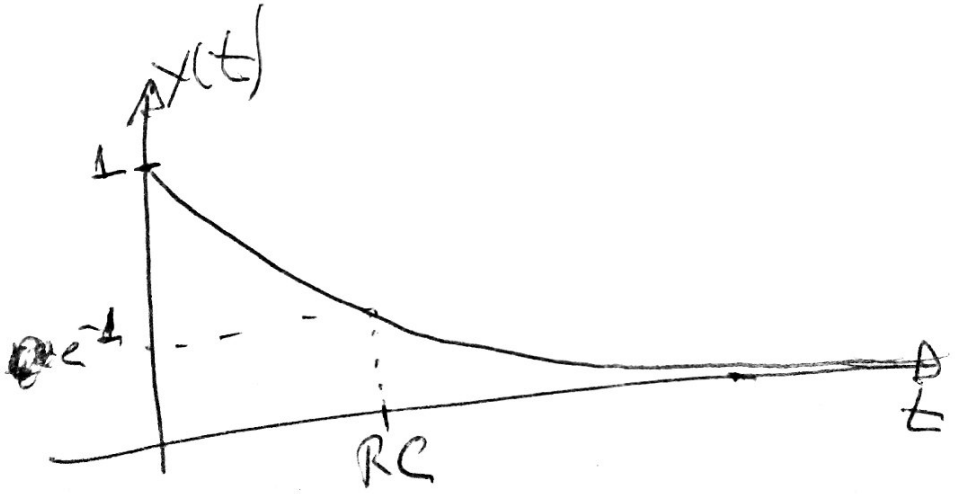
Resposta Forçada (reg. permanente) p/ entrada de grau ($u(t) = 1/4(t)$)

$$\frac{dy(t)}{dt} = -\frac{1}{RC} y(t) + \frac{du}{dt}$$

$$y_p = \frac{1}{R}\delta(t) \rightarrow y_f = 0$$

$$\int_{0_-}^{0_+} \frac{dy}{dt}\, dt = \int_{0_-}^{0_+} -\frac{1}{RC} \cancel{y}^{\,0}\, dt + \int_{0_-}^{0_+} \frac{du}{dt}\, dt$$

$y(0_+) = 1$ $\rightarrow$ Um impulso na entrada altera as condições iniciais



Massa-mola

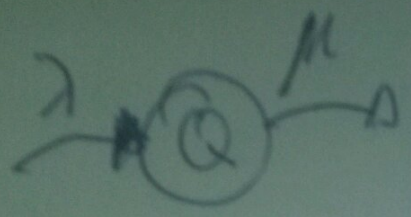
$$Y_f(t) = u(t) + \frac{b}{k} \cdot \frac{du}{dt}$$

Para $u(t) = H(t)$

$$Y_f(t) = u(t)$$

$$\dot{x}_1 = \frac{k}{m} x_1 + \frac{b}{m} x_2 + u(t)$$

Servidor 1



$u = \mu(t)$

$y = Q(t)$

$$\mu = \frac{1}{T} Q$$

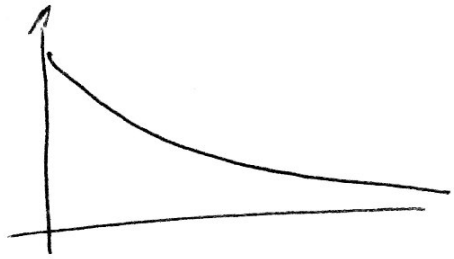
$$\frac{dQ}{dt} = -\frac{1}{T} Q(t) + \mu(t)$$

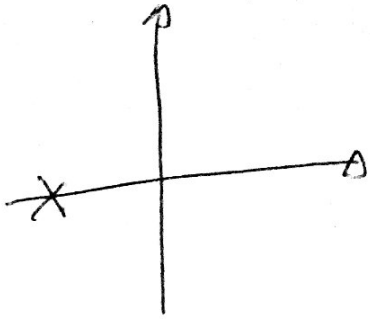
Casos de autovalores:

Real:

$a < 0$

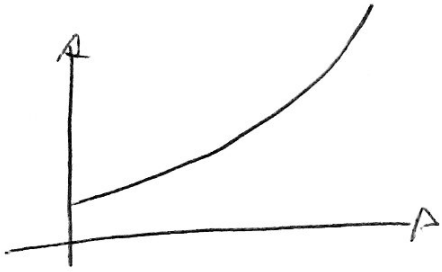
$z(t) = e^{at}$

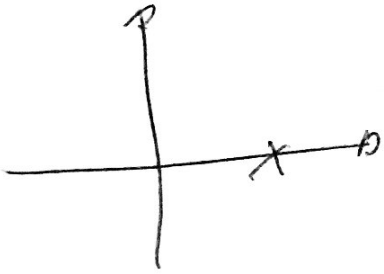




$a > 0$.

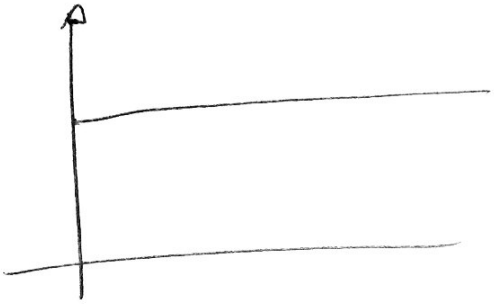
$z = e^{at}$

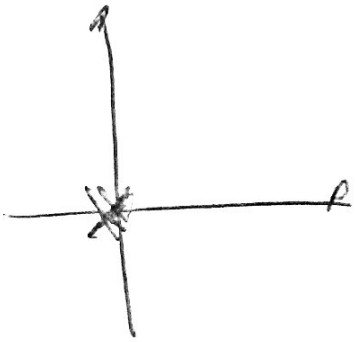




$a = 0$

$z = e^{0t} = 1$



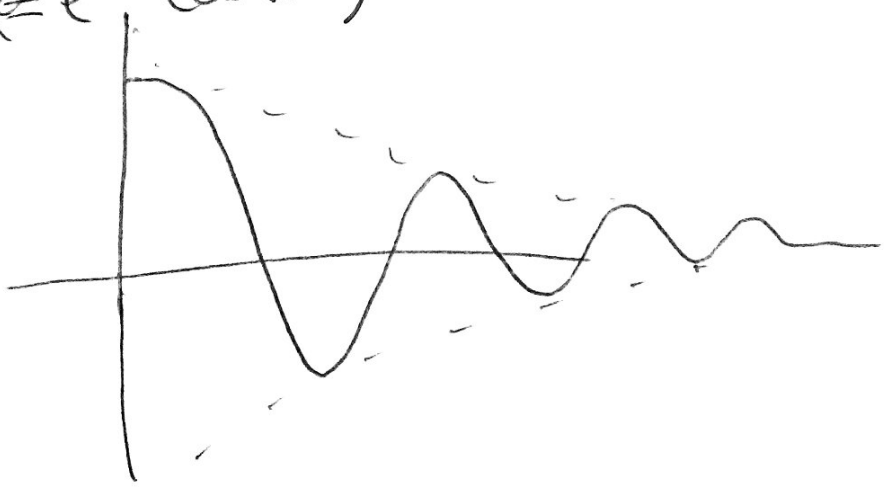


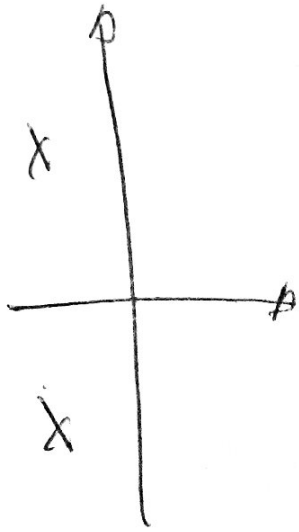
Complexo:

$\lambda = a \pm jb$

$a < 0$

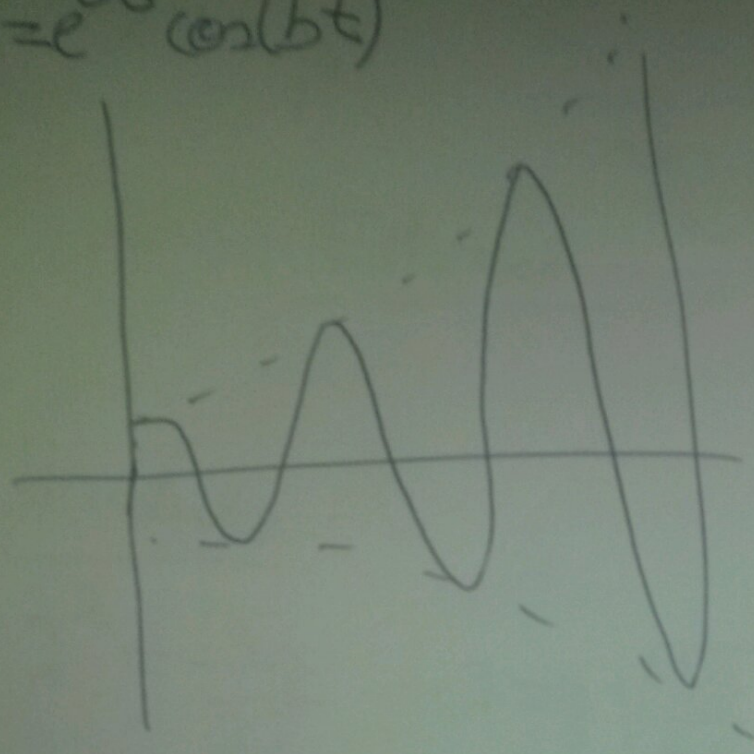
$z(t) = e^{at} \cos(bt)$

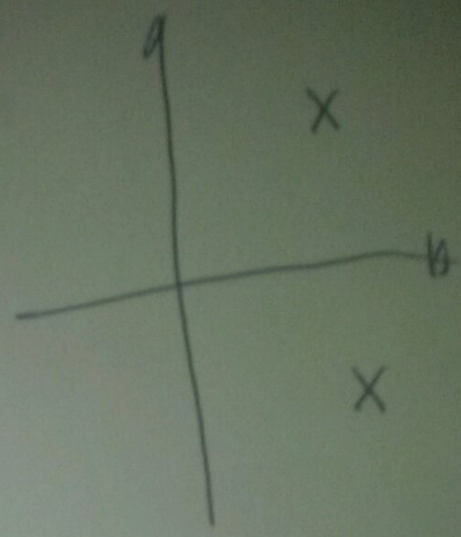




$a > 0$

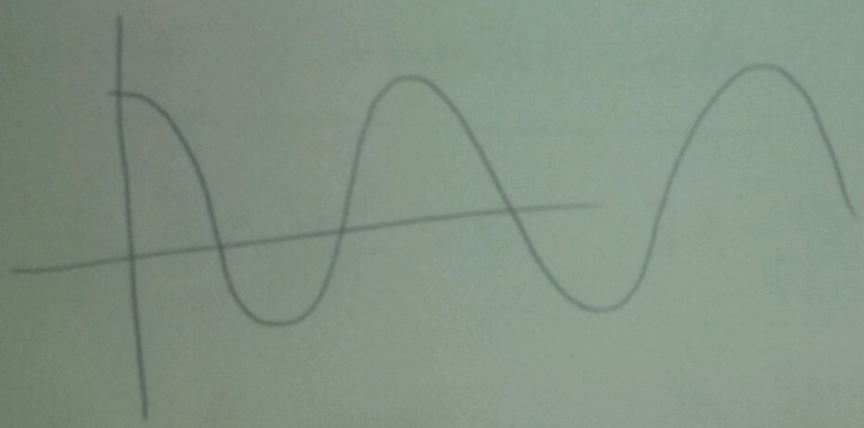
$z(t) = e^{at}\cos(bt)$

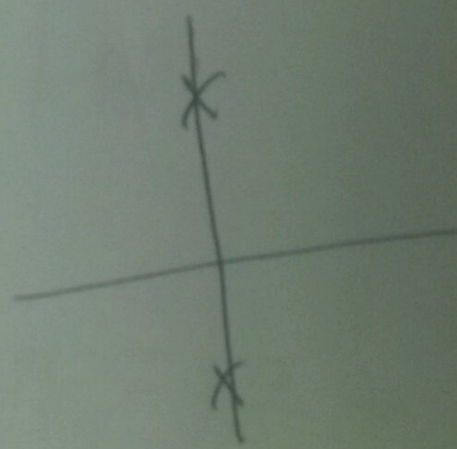




$a = 0$

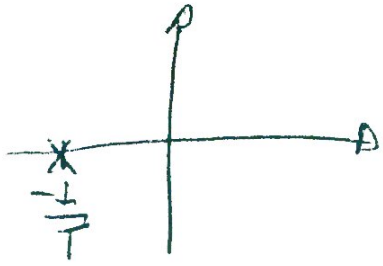
$z(t) = \cos(bt)$





Servidor:

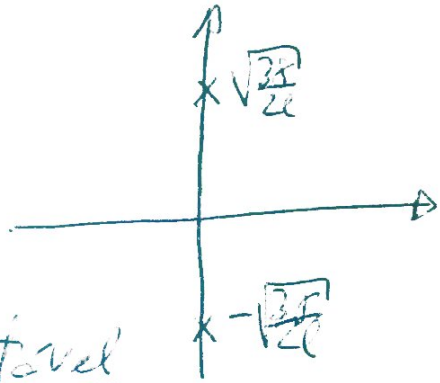
$$\lambda = -\frac{1}{T}$$



Estável

Pêndulo:

$$\lambda^2 = -\frac{3g}{2l} \qquad \lambda = \pm j\sqrt{\frac{3g}{2l}}$$



Marginalmente estável

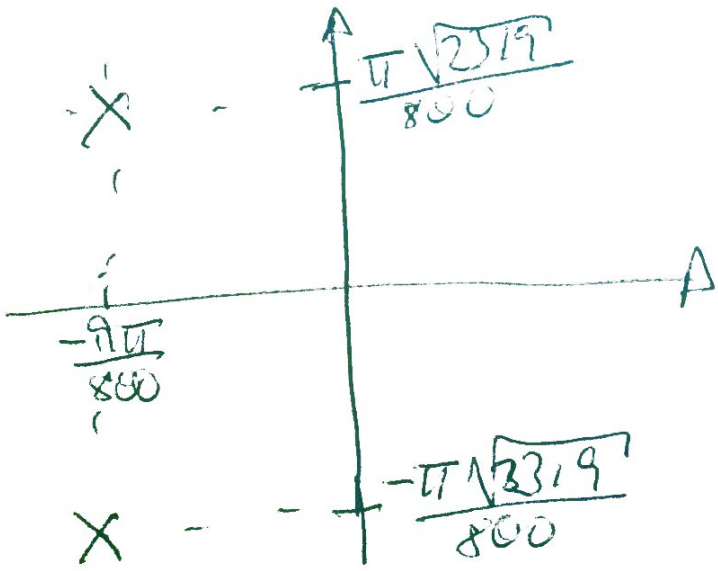
Pêndulo c/ amortecimento

$$\lambda^2 + \frac{9\pi}{400}\lambda + \frac{3\pi^2}{800} = 0 \qquad \lambda = \frac{-\frac{9\pi}{400} \pm \sqrt{\frac{81\pi^2}{160000} - \frac{12\pi^2}{800}}}{2} =$$

$$= -\frac{9\pi}{800} \pm \frac{1}{2}\sqrt{\frac{81\pi^2 - 2400\pi^2}{160000}} =$$

$$= -\frac{9\pi}{800} \pm \frac{1}{800}\sqrt{-2319\pi^2} =$$

$$= -\frac{9\pi}{800} \pm \frac{j\pi}{800}\sqrt{2319}$$



Estável

Circuito integrador:

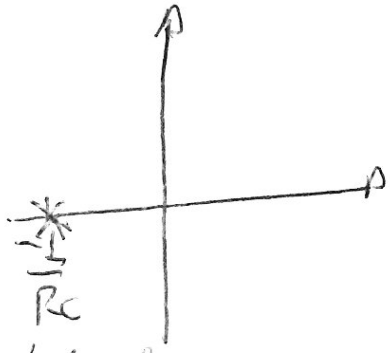
$$i = C\frac{dy}{dt}$$

$$C\frac{d^2y}{dt^2} = -\frac{1}{R}\frac{dy}{dt} + \frac{1}{R}\frac{du}{dt}$$

$$\frac{d^2y}{dt^2} = -\frac{1}{RC}\frac{dy}{dt} + \frac{1}{RC}\frac{du}{dt}$$

$$\frac{dy}{dt} = -\frac{1}{RC}y + \frac{1}{RC}u$$

$$\lambda = -\frac{1}{RC}$$



Estável

Motor DC

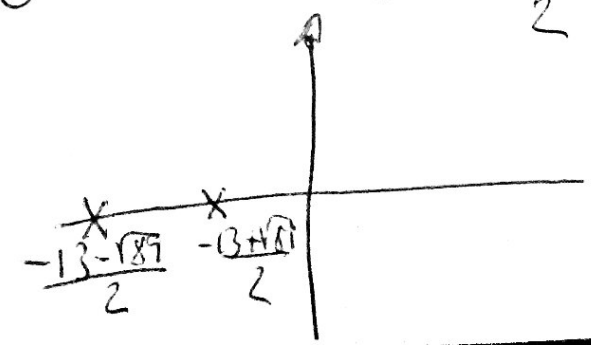
$$\det\begin{bmatrix} -\frac{R_a}{L_a} - \lambda & \frac{K_b}{L_a} \\ \frac{K}{J} & -\frac{b}{J} - \lambda \end{bmatrix} = 0$$

$$\frac{R_a b}{L_a J} + \frac{R_a}{L_a}\lambda + \frac{b}{J}\lambda + \lambda^2 - \frac{K K_b}{J L_a} = 0$$

$$\lambda^2 + \left(\frac{R_a}{L_a} + \frac{b}{J}\right)\lambda + \frac{R_a b - K K_b}{L_a J} = 0$$

$$\lambda^2 + (10+3)\lambda + \frac{1}{2}\,\frac{3}{100}\cdot 20\cdot 100 - \frac{1}{20}\cdot\frac{1}{20}\cdot 20\cdot 100 = 0$$

$$\lambda^2 + 13\lambda + 20 = 0 \qquad \lambda = \frac{-13 \pm \sqrt{169-80}}{2} = \frac{-13 \pm \sqrt{89}}{2} =$$

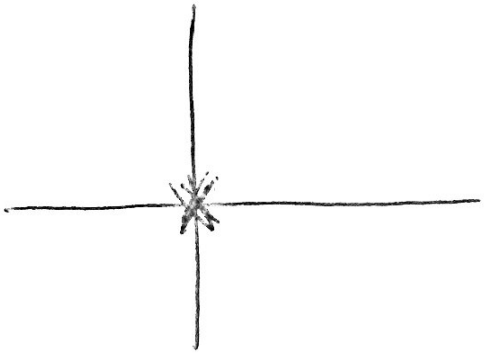


Estável

Satélite:

$\lambda^2 = 0$ $\quad \lambda_1 = 0 \rightarrow z_1(t) = 1$

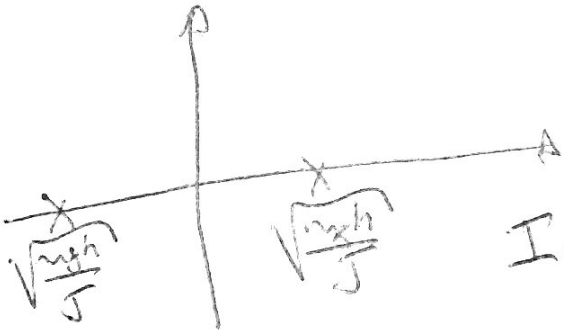
$\lambda_2 = 0 \rightarrow z_2(t) = t$



instável

Pêndulo invertido

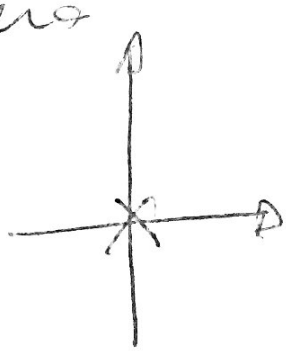
$\lambda^2 = \frac{m g h_0}{J}$ $\quad \lambda = \pm\sqrt{\frac{m g h_0}{J}}$



Instável

Caixa d'água

$\lambda = 0$



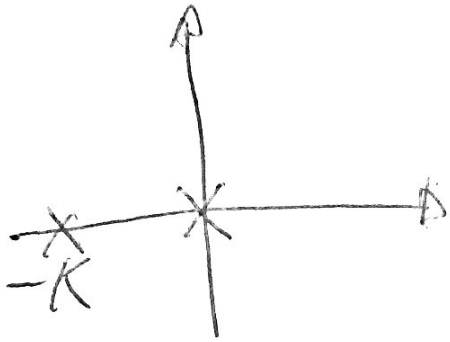
marginalmente estável

Linha de Montagem

$$\det\begin{bmatrix} -\lambda & K \\ 0 & -K-\lambda \end{bmatrix} = 0 \qquad K\lambda + \lambda^2 = 0$$

$$\lambda_1 = 0 \qquad \lambda_2 = -K$$

Marginalmente estável



Imunitação

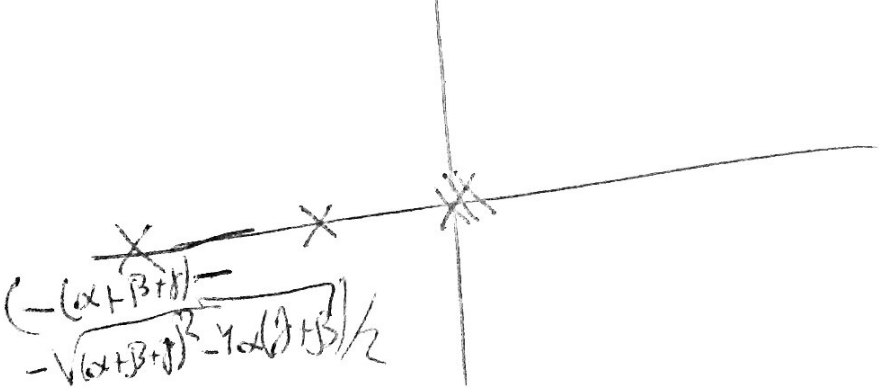
$$\det\begin{bmatrix} -\alpha-\lambda & 0 & 0 & 0 \\ \alpha & -(\gamma+\beta)-\lambda & 0 & 0 \\ 0 & \gamma & -\lambda & 0 \\ 0 & \beta & 0 & -\lambda \end{bmatrix} = 0$$

$$\lambda^2(\alpha\gamma + \alpha\beta + \alpha\lambda + (\gamma+\beta)\lambda^2) = 0$$

$$\lambda_1 = 0 \qquad \lambda_2 = 0$$

$$\lambda^2 + (\alpha+\beta+\gamma)\lambda + \alpha\gamma + \alpha\beta = 0$$

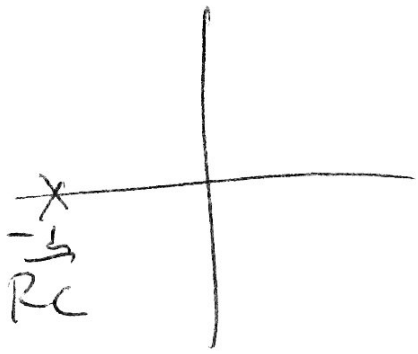
$$\lambda = \frac{-(\alpha+\beta+\gamma) \pm \sqrt{(\alpha+\beta+\gamma)^2 - 4\alpha\gamma - 4}}{2}$$



Instável

Diferenciador

$$\lambda = \frac{-1}{RC}$$

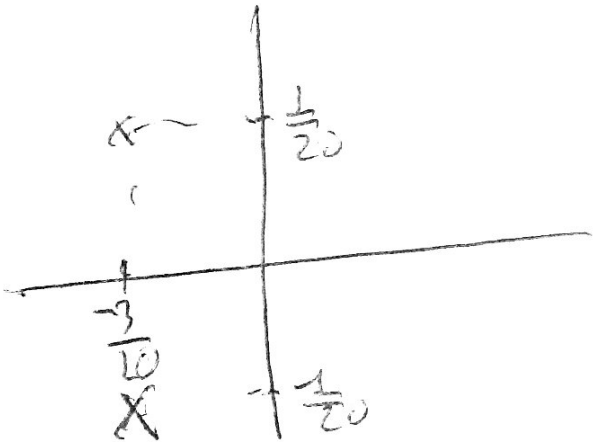


Estável

Massa-mola-amortecedor

$$\lambda^2 + \frac{b}{m}\lambda + \frac{K}{m} = 0 \qquad \lambda = \frac{-b}{m} \pm \sqrt{\frac{b^2}{m^2} - \frac{4K}{m}}$$

$$\lambda = \frac{-0,3 \pm \sqrt{\frac{9}{100} - \frac{40}{100}}}{2} = \frac{-3}{20} \pm j\frac{1}{20}$$



Estável